Quicksort Improvements - 58 Years Later

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This presentation is mostly based on the project report:

Quicksort Improvements - 57 Years Later

By Konrad Rafał Witaszczyk and Pavel Kucera

Quicksort

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 if p < r
   q = PARTITION(A, p, r)
   QUICKSORT(A, p, q - 1)
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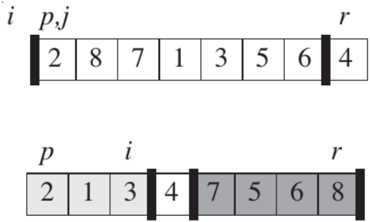

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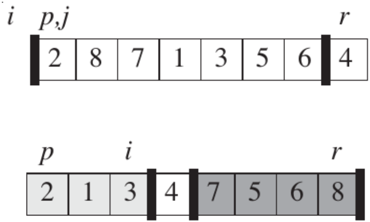

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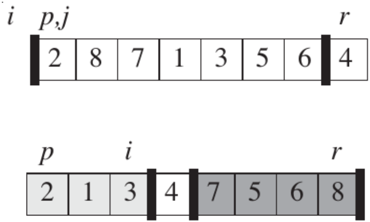

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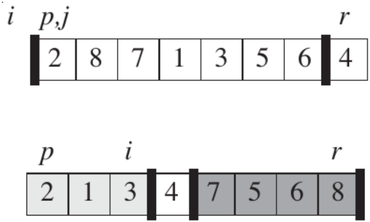

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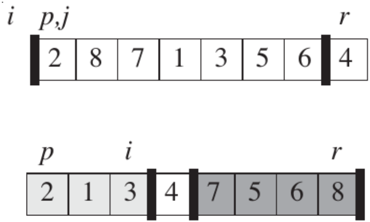

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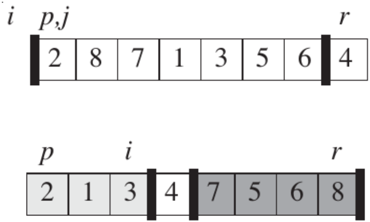

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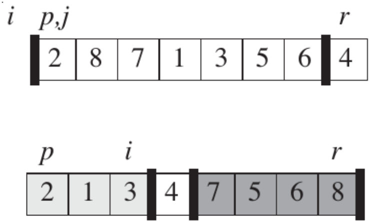

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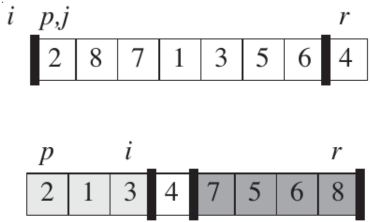

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- Worst-case running time: $O(n^2)$;
- Expected running time: $O(nlgn)$;
- Constant in $O(nlgn)$ is quite small;
- It's an in-place algorithm;
- It's an unstable algorithm.

Motivation

- Inspired by the paper 'Heap Construction – 50 Years Later', Stefan Edelkamp, Amr Elmasry, and Jyrki Katajainen;
- There are many algorithms that improve the original Quicksort algorithm;
- Improving an algorithm for one property can introduce additional penalty in other metrics, for example branch mispredictions and number of element moves;
- We'd like to verify statements from scientific papers about proposed algorithms;
- We'd like to measure number of comparisons, moves, cache misses and branch mispredictions and see if an algorithm is optimal for all these metrics;
- Is there any framework to measure performance in these terms that would be accurate, portable and easy to use?

Selected algorithms

1. Hoare's Quicksort.
2. Tuned Quicksort.
   Median-of-three pivot selection.
3. Instrosort.
   Quicksort with heapsort after $2 * logn$ recursion depth.
4. Skewed Introsort.
   Introsort with a skewed pivot.
5. Super Scalar Samplesort.
   Faster than `std::sort` in many cases but uses 2-3x additional memory.
6. In-place Parallel Super Scalar Samplesort.
   Cache-efficient, avoids branch-mispredictions.
7. Standard C++ library.

Goal

What is a running time, a number of comparisons, element moves, branch mispredictions and cache misses of each algorithm?

Performance tools

During the project we tried the following tools and techniques:

- DTrace;
- pmcstat;
- Processor Counter Monitor (PCM);
- Instruments (Xray);
- Instrumentation;
- Simulation.

Accuracy vs real case metrics

In order to measure resource usage in parts of any program we must refer to them using assembly symbols.

However, in order to run a program as it was used in a production environment we must use compiler optimizations that remove assembly symbols. In this case we can still measure performance relative to some baseline and hope that it was enough accurate.

Name mangling

Encoding function prototypes into unique names.

It is used by a compiler (for example for overloading) and a linker. Each compiler implements its own name mangling algorithm, e.g. for GCC 8.2.0 we have:

```
void hoare::sort<int *, std::less<int> >(int *, int *, std::less<int>);
_ZN5hoare4sortIPiSt4lessIiEEEvT_S4_T0_
```

Name mangling: utils/demangle and utils/findsymbol

Using GCC ABI (`abi::__cxa_demangle`), `demangle` prints a function prototype for a given symbol:

```
$ ./utils/demangle _ZN5hoare4sortIPiSt4lessIiEEEvT_S4_T0_
void hoare::sort<int*, std::less<int> >(int*, int*, std::less<int>)
$
```

Using `nm` and `demangle`, `findsymbol` prints all symbols corresponding to function prototypes matching a regular expression in a binary file:

```
$ ./utils/findsymbol.sh ./hoare 'Element<int>::operator=\(Element<int> const↩
     &\)'
_ZN7ElementIiEaSERKS0_
$
```

DTrace: running time (benchmark/time.d)

```
uint64_t total;
self uint64_t depth, start;

BEGIN
{
  self->depth = 0;
  total = 0;
}

pid$target::$1:entry
/ self->depth == 0 /
{
  self->start = vtimestamp;
}

pid$target::$1:entry
{
  self->depth = self->depth + 1;
}

pid$target::$1:return
{
  self->depth = self->depth - 1;
}

pid$target::$1:return
/ self->depth == 0 /
{
  total = (total + vtimestamp - self->start);
}

END
{
  printf("%u", total / 1000);
}
```

DTrace: running time (benchmark/time.d)

For `hoare::sort` we can execute:

```
$ sudo dtrace -s benchmark/time.d -c './hoare input.txt' \
    _ZN5hoare4sortIPiSt4lessIiEEEvT_S4_T0_
82695
$
```

DTrace: number of moves (benchmark/count.d)

```
uint64_t ncalls;

BEGIN
{
        ncalls = 0;
}

pid$target::$1:entry
{
        ncalls = ncalls + 1;
}

END
{
        printf("%u", ncalls);
}
```

DTrace: number of moves (benchmark/count.d)

For `Element<int>::operator=(Element<int> const&)` we can execute:

```
$ sudo dtrace -s benchmark/count.d -c './hoare input.txt' ↩
    _ZN7ElementIiEaSERKS0_
90897
$
```

DTrace: branch mispredictions and cache misses

DTrace for Solaris includes `cpc` provider that implements probes for CPU performance counters. Unfortunately, the provider has not been ported to FreeBSD, macOS or Linux.

pmcstat

`pmcstat` is a performance measurement utility on FreeBSD that gives access to CPU counters via `hwpmc` driver, including:

```
# pmccontrol -L
...
        BR_INST_RETIRED.ALL_BRANCHES
...
        MEM_LOAD_UOPS_RETIRED.L1_MISS
        MEM_LOAD_UOPS_RETIRED.L2_MISS
...
```

pmcstat

We use Intel Core i7-3610QM CPU. In Intel 64 and IA-32 Architectures Software Developer's Manual, Volume 3 we find the meaning of the counters:

- `BR_MISP_RETIRED.ALL_BRANCHES` – mispredicted branch instructions at retirement;
- `MEM_LOAD_UOPS_RETIRED.L1_MISS` – retired load uops whose data source followed an L1 miss;
- `MEM_LOAD_UOPS_RETIRED.L2_MISS` – retired load uops that missed L2, excluding unknown sources.

pmcstat

We run `pmcstat` in counting and sampling modes for a user process and later calculate results:

```
$ pmcstat -S BR_MISP_RETIRED.ALL_BRANCHES -P BR_MISP_RETIRED.ALL_BRANCHES \
    -O hoare.pmcstat ./hoare input.txt
$ pmcstat -R hoare.pmcstat -G -
...
07.08%  [1069] _ZN5hoareL9partitionIPiSt4lessIiEEET_S4_S4_T0_ @
    /ztank/priv/KU/AE/project/src/hoare
...
```

According to `pmcstat` there were 1069 branch mispredictions in the partition function of the original Quicksort implementation. It's 7.08% of all branch mispredictions that occurred in the program.

Instrumentation

As mentioned before, explained methods does not allow us to use compiler optimizations. We decided to introduce code instrumentation and use the `-O3` optimization level.

Instrumentation

We introduce the following counters:

```
#ifdef MEASURE_COMPARISONS
static uint64_t ncomparisons = 0;
#endif

#ifdef MEASURE_MOVES
static uint64_t nmoves = 0;
#endif

#ifdef MEASURE_TIME
static struct timespec dtime;
#endif
```

Instrumentation

In case of measuring comparisons or moves we wrap elements by a class `Element` in which we define:

```
#ifdef MEASURE_MOVES
    Element(Element<T> const &element) {
      *this = element;
    }

    Element& operator=(Element const &element) {
      this->value = element.value;
      nmoves++;
      return (*this);
    }
#endif

    friend bool operator<(const Element<T> &x,
        const Element<T> &y) {
#ifdef MEASURE_COMPARISONS
      ncomparisons++;
#endif
      return (x.value < y.value);
    }

    friend bool operator==(const Element<T> &x, const Element<T> &y) {
#ifdef MEASURE_COMPARISONS
      ncomparisons++;
#endif
      return (x.value == y.value);
    }
```

Instrumentation

In case of measuring running time we calculate a difference between the time after calling and the time before calling `sort`:

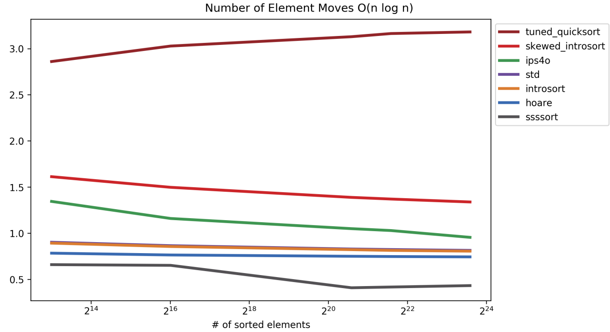
```
#ifdef MEASURE_TIME
  clock_gettime(CLOCK_MONOTONIC, &start);
#endif

  NAME::sort(first, last + 1, std::less<V>());

#ifdef MEASURE_TIME
  clock_gettime(CLOCK_MONOTONIC, &dtime);
  dtime.tv_nsec -= start.tv_nsec;
  dtime.tv_sec -= start.tv_sec;
  if (dtime.tv_nsec < 0) {
    dtime.tv_sec--;
    dtime.tv_nsec += 1000000000;
  }
#endif
```

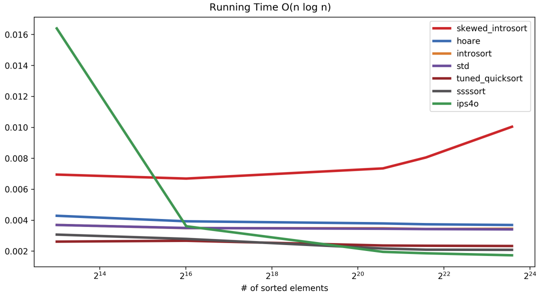
Simulation

Valgrind provides Cachegrind which can simulate a machine running a program and give a number of cache misses and branch mispredictions.

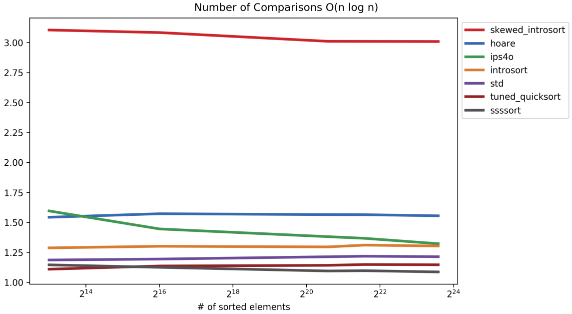
However, it doesn't consider other activity, including kernel, other processes, TLB misses.

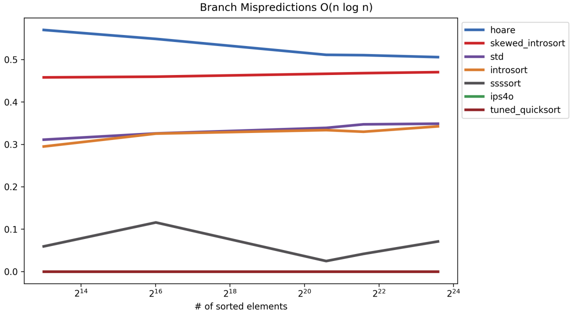
Results: tuned_quicksort is ugly



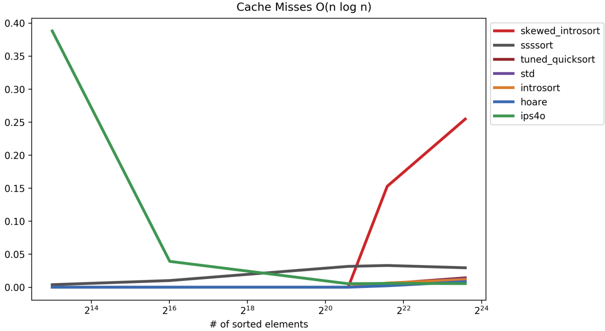


Results: skewed_introsort is bad

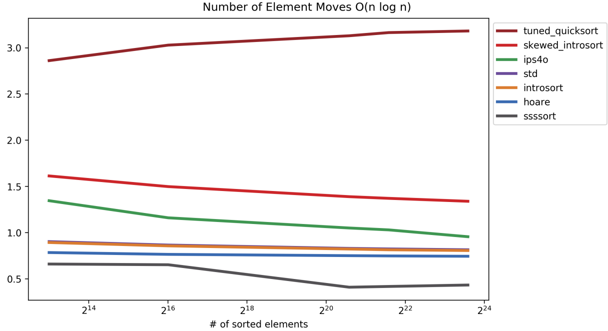




Results: ssssort is good



Results: `std::sort` uses `introsort`



Conclusion

- We managed to try a lot of tools that we can use in many other areas;
- We found two implementations that give very good results and we proved that with experiments;
- Creating a portable framework for performance measurements would be a very useful project.

Thank you for your attention!

ask questions